Ultimate Burr Set

The Ultimate Burr Set from Creative Crafthouse (www.creativecrafthouse.com) is based on the burr set documented in *Creative Puzzles of the World* by Van Delft and Botermans (1978). The set consists of 27 pieces (all unique) that can be used 6 at a time to assemble a burr puzzle.

Figure 1. Ultimate Burr Set

The design for each piece is provided in Table 1. The Burr ID Tool[1] on Rob Stegmann's Interlocking Puzzle webpage was used to generate the piece designs. The tool generates a unique ID for each possible burr piece based on a numbering system developed by Jurg von Kanel[2].

Table 1. Ultimate Burr Set Pieces

Piece ID	Piece Design	Piece ID	Piece Design	Piece ID	Piece Design
0	1	9	256	18	444
1	60	10	928	19	506
2	410	11	508	20	416
3	414	12	448	21	476
4	442	13	480	22	124
5	1024	14	224	23	412
6	992	15	128	24	64
7	960	16	478	25	188
8	512	17	192	26	474

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[1] http://home.comcast.net/~stegmann/burrpcid.htm
[2] http://www.research.ibm.com/BurrPuzzles/Burr6.html

6 Piece Burr Puzzles

Delft and Botermans provide the following definition for a burr puzzle:

> *Burr puzzles, also called Chinese cross puzzles and notched-stick puzzles, are probably the construction puzzles that are most well known to enthusiasts. They consist of a symmetrical crosslike structure made up of three pairs of interlocking wooden bars. Each pair of bars is at right angles to the other two pairs, and they are arranged so that there is no empty space in the center of the puzzle. The bars, which originally are blocks of equal length with a square cross section, have a number of cutouts which are made by removing cube-shaped volumes of wood, each with sides equal to half the width of the bar.[3]*

The standard burr puzzle consists of 6 pieces (bars) that are assembled as shown in Figure 2. If Figure 2 is examined closed, it can be seen that each piece consists of smaller cubes. Most common burr puzzles are made from 2 x 2 stock and the length may vary with lengths 6 and 8 being the most common. In the case of the Ultimate Burr Set and as shown in Figure 2, the size of each piece is 2 x 2 x 8. Various different burr puzzles can be made by varying the cubes that are removed from each piece to allow them to interlock with each other.
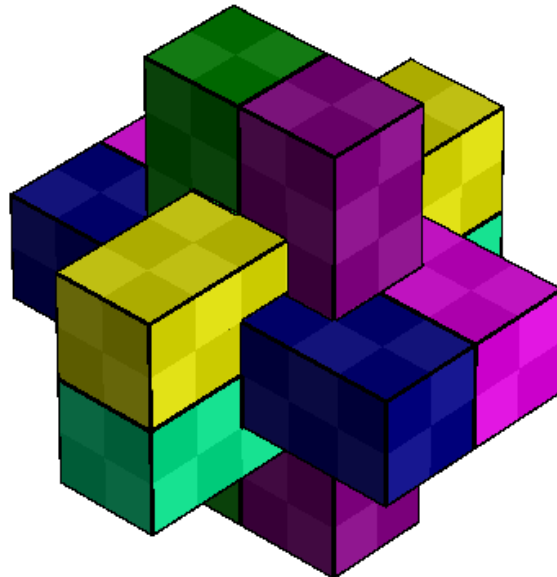

Figure 2. Burr Puzzle Image from BurrTools[4]

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[3] Van Delft and Botermans, Creative Puzzles of the World, 1978

[4] BurrTools is an amazing and free program developed and maintained by Andreas Röver. It is ideal for designing and analyzing puzzles that can be constructed from cubes as well as a few other elementary shapes. The latest version of BurrTools can be downloaded from: http://burrtools.sourceforge.net. Version 0.5.2 is highly recommended.

The different possible pieces and puzzles have been extensively analyzed and the following websites are just a sampling of many that provide information on 6 piece burr puzzles:

- *www.research.ibm.com/BurrPuzzles* - This site provided by IBM has a good overview of burr puzzles and the 6 piece burr puzzle in particular.

- *home.comcast.net/~billcutler/docs* - Bill Cutler started performing analyses on 6 piece burrs in the 70's. This site includes documentation on some of Bill Cutler's research. Burr and other puzzles can also be purchased via the links to other pages.

- *home.comcast.net/~stegmann/interlocking.htm* - This is Rob Stegmann's page on interlocking puzzles, which starts with a thorough description of 6 piece burr puzzles. It catalogs the pieces and identifies which pieces are included in well known puzzles. The site includes numerous pictures of puzzles that have been created and sold around the world.

Ultimate Burr Set

Van Delft and Botermans provides the pieces and their orientations for creating 69, 6 piece burrs that have no voids/holes in the center of the burr. 48 of these solutions have all unique pieces and 21 require 2 pieces with the same shape. The Ultimate Burr Set can be used to build the 48 puzzles from *Creative Puzzles of the World* that have unique pieces (i.e., don't require duplicate pieces) and includes the solution for each.

Analysis with BurrTools indicates that there are 59 solutions for solid burrs using the set of 27 unique pieces. Of these 59 solutions, 3 of them use the same pieces as another solution, so there are only 56 unique piece sets with 3 of them that can be assembled 2 different ways. The additional 8 sets beyond the 48 solutions provided with the Ultimate Burr Set use the following pieces:

1. 7,5,14,9,0,25 (2 solutions)
2. 6,5,9,17,0,25 (2 solutions)
3. 7,6,14,9,0,17 (2 solutions)
4. 10,5,17,9,0,14
5. 10,5,9,14,0,17
6. 6,5,17,25,0,9
7. 7,5,25,14,0,9
8. 10,5,17,14,0,9

Solid constructions require that the last key piece be the solid piece (piece 0), which can be inserted and removed in one move. However, most interesting (and more difficult) burr puzzles utilize constructions that have holes in the center that are not visible when the puzzle is constructed. Allowing constructions with holes in the center allows for configurations where multiple moves are required to insert the last piece. This allows the final puzzle to be more complex and difficult to solve.

An analysis of the Ultimate Burr Set with the BurrTools program without the solid piece and allowing for voids in the center resulted in the following statistics:

- 1,104,397 total solutions (out of 230,230 x 15,728,640 = 3,621,204,787,200. Refer to the next two bullets for how these numbers are derived.)

- 18,067 unique piece sets (out of 26! / [(26 – 6)! x 6!] = 230,230).

- 1,969 max solutions using same 6 pieces (out of 5! x 8^5 x 4 = 15,728,640 possible solutions). This is with pieces 5, 6, 7, 8, 12, and 13. Two of these solutions require 7 moves to take the first piece out. However, 928 of them can have the first piece removed in one move.

- 535 solutions use pieces that can't be rearranged into another solution, i.e., the solution is unique for that combination of pieces.

The Unique Solutions

The objective of this analysis was to derive a more difficult set of puzzles with unique solutions that could be constructed from the Ultimate Burr Set. Of the 535 unique solutions, some are more difficult than others to construct. As a basic measure of difficulty, the number of moves required to add the last piece will be used as a proxy for the difficulty. There are many cases where the last piece does not go in by itself but is inserted as an ensemble with other pieces. The 535 solutions fall into the following levels of difficulty:

Moves to Add Last Piece	Solutions
1	256
2	127
3	75
4	45
5	32

For your enjoyment, the following tables provide the pieces required to make each unique solution categorized by the number of moves required to add the last piece.

Table 2. Solutions Requiring 1 Move To Add Last Piece

7,10,13,17,19,22	1,6,7,9,18,22	1,5,9,10,22,25	1,5,7,14,23,24	1,6,9,10,15,25	1,6,7,14,16,24
7,9,10,15,19,24	7,9,10,15,19,25	5,10,17,18,19,22	1,6,7,17,21,24	3,5,10,14,15,22	3,5,10,14,15,24
1,6,10,12,14,24	1,5,6,16,20,24	4,6,7,12,15,22	3,6,10,14,15,24	1,5,6,12,16,24	1,5,10,14,20,24
1,5,7,18,20,22	1,5,7,14,16,22	7,10,15,18,22,25	1,6,7,14,24,25	1,2,5,7,14,15	2,5,10,15,17,22
1,5,10,12,22,25	1,2,5,6,15,17	7,9,10,11,23,24	6,7,9,21,22,24	6,9,10,15,22,25	1,5,6,12,24,25
5,10,16,17,19,22	7,9,10,22,24,25	6,10,13,15,19,24	1,5,6,9,18,22	1,5,10,17,18,22	6,10,16,17,22,24
7,10,14,15,16,20	4,6,10,12,15,22	1,6,9,10,16,24	1,4,6,7,14,15	1,6,7,17,22,25	7,9,10,15,20,24
7,9,10,15,20,25	1,3,6,7,9,11	1,3,6,7,9,14	1,3,6,7,9,15	5,10,17,18,20,22	7,10,14,17,22,25
1,6,10,12,15,18	1,6,10,12,15,25	3,6,7,15,17,21	4,6,7,15,21,24	1,7,10,12,15,21	1,5,7,20,22,25
6,10,15,18,20,24	7,10,14,15,24,26	3,6,9,10,17,22	1,5,9,10,16,24	1,6,9,10,24,25	6,9,10,16,22,24
1,5,7,13,18,22	7,9,10,11,24,26	3,5,6,16,22,24	7,9,10,17,19,22	7,10,11,17,18,22	7,10,11,22,24,25
4,6,10,13,15,24	5,10,16,17,20,22	1,6,7,15,17,26	1,6,7,9,20,22	1,5,9,10,24,25	6,7,15,17,22,23
6,7,15,17,22,26	7,10,13,15,23,24	7,9,10,16,20,22	4,6,9,10,14,24	4,6,7,14,15,22	4,6,7,14,15,24
7,10,14,18,22,24	1,3,5,7,14,15	1,3,5,7,14,22	5,10,17,20,22,24	5,10,17,20,22,25	3,6,7,9,20,22
1,7,10,13,15,16	1,7,10,13,15,19	1,7,10,13,15,25	4,6,7,9,20,22	4,6,7,15,22,25	1,6,10,11,17,24
7,10,13,15,16,20	1,7,9,10,18,22	7,10,13,20,22,24	7,10,12,15,22,23	7,10,12,15,22,26	3,5,6,17,22,24
5,7,22,23,24,25	2,5,10,11,15,24	7,10,11,17,19,22	6,7,15,20,23,24	4,5,7,15,22,25	1,5,10,14,15,20
1,6,7,9,21,24	6,10,17,22,24,25	7,10,14,15,18,19	1,5,10,17,20,22	7,10,13,15,24,26	4,6,9,10,15,22
4,6,9,10,15,24	4,6,9,10,15,25	1,5,10,13,16,24	2,5,7,14,15,22	1,7,10,14,15,16	1,7,10,14,15,19
3,6,9,10,11,24	1,7,10,14,15,25	7,9,10,15,22,23	7,9,10,15,22,26	3,6,7,9,21,24	6,10,12,22,24,25
7,9,10,14,23,24	4,6,7,9,21,24	2,5,7,15,22,25	1,5,9,10,18,22	1,5,10,13,24,25	7,10,13,18,22,24
6,10,15,16,24,25	5,7,14,22,23,24	7,9,10,18,20,22	1,5,7,9,16,24	1,5,10,14,16,24	1,5,6,15,17,23
1,6,7,9,22,25	7,10,14,15,19,25	5,10,17,19,22,25	7,9,10,15,23,24	7,9,10,15,23,25	1,5,6,17,23,24
6,10,12,17,19,22	1,5,10,12,18,22	1,3,5,6,17,22	1,7,10,13,17,22	7,9,10,14,24,26	1,5,7,9,24,25
1,5,10,14,24,25	3,6,7,12,15,21	3,6,7,12,15,22	7,10,15,16,20,22	3,6,7,15,20,22	1,6,7,9,15,23
1,6,7,9,15,26	7,10,13,15,18,19	1,6,7,17,18,22	1,7,9,10,20,22	3,6,7,9,15,22	3,6,7,9,15,24
7,9,10,20,22,24	7,9,10,20,22,25	7,10,14,17,18,22	7,10,14,22,24,25	4,6,7,9,15,22	4,6,7,9,15,24
5,10,17,22,23,24	7,9,10,17,22,23	7,9,10,17,22,26	1,3,6,9,10,15	4,6,7,15,17,22	1,5,6,9,22,25
6,7,12,15,22,23	1,5,6,20,24,25	1,5,10,17,22,25	7,10,12,21,22,24	4,6,9,10,17,22	5,7,18,22,23,24
1,6,7,11,16,24	1,2,5,7,11,15	7,9,10,15,24,25	7,9,10,15,24,26	3,6,7,13,15,22	3,6,7,13,15,24
6,10,12,17,20,22	3,6,10,11,15,24	2,5,10,14,15,24	4,5,7,15,17,22	7,10,12,13,19,22	1,5,10,11,20,24
5,10,15,17,22,23	1,6,7,9,16,24	1,5,7,11,16,22	1,6,7,11,24,25	4,5,10,15,17,22	7,10,13,15,19,25
7,9,10,21,22,24	1,5,7,14,21,24	4,6,10,15,17,22	1,7,9,10,21,24	1,5,7,13,22,25	7,10,14,17,19,22
1,4,6,7,11,15	7,9,10,18,22,24	6,10,14,16,17,24	6,7,9,15,22,23	6,7,9,15,22,26	1,6,10,15,17,18
1,6,10,15,17,25	7,10,11,17,22,25	4,6,7,15,18,22	2,5,7,15,17,22	1,6,7,9,24,25	1,7,10,15,17,21
6,9,10,22,24,25	7,10,11,12,19,22	6,10,14,17,24,25	3,6,9,10,14,24	3,6,7,14,15,24	7,9,10,15,25,26
7,10,11,15,24,26	3,5,6,22,24,25	4,6,7,10,15,22	3,6,10,12,15,22	6,7,17,19,20,22	7,10,12,14,19,22
7,10,13,22,24,25	4,5,7,15,18,22	6,10,12,16,22,24	6,7,12,19,20,22	1,6,7,9,17,23	1,6,7,9,17,26
1,4,6,9,10,15	5,7,17,22,23,24	7,10,13,15,20,24	4,6,9,10,11,24	1,7,9,10,22,25	1,5,6,17,18,24
1,3,5,7,11,15	1,3,5,7,11,22	7,10,11,18,22,24	2,5,7,15,18,22	3,6,10,13,15,24	6,10,12,20,22,24
3,6,9,10,15,22	3,6,9,10,15,24	3,6,9,10,15,25	7,10,12,15,19,22	1,5,7,14,15,26	6,7,17,21,22,24
1,7,9,10,15,19	1,7,9,10,15,20	1,7,9,10,15,23	1,7,9,10,15,25	1,7,9,10,15,26	1,4,6,7,9,11
1,4,6,7,9,14	1,4,6,7,9,15	7,10,15,17,22,23	3,5,10,14,22,24		

Table 3. Solutions Requiring 2 Moves To Add Last Piece

6,7,14,19,24,25	1,5,7,10,19,22	7,10,11,15,18,20	1,3,6,7,8,22	3,5,10,14,15,25	6,9,10,18,19,24
1,3,5,6,13,24	4,5,6,14,19,24	3,6,7,14,17,22	1,7,10,11,15,20	6,10,15,17,20,22	6,10,15,17,20,25
1,4,5,8,10,22	6,7,14,16,20,24	5,10,15,18,24,26	7,8,10,16,19,22	1,6,7,9,19,24	1,6,7,15,16,19
1,6,7,15,16,20	5,9,10,16,24,26	5,10,14,19,22,24	4,6,7,13,15,20	6,7,13,16,19,24	1,3,5,6,14,24
6,7,13,19,24,25	1,6,10,12,15,20	6,7,14,18,19,24	4,6,7,11,17,22	6,7,9,19,22,24	5,9,10,19,22,24
1,4,5,6,13,24	4,6,7,14,15,20	6,7,15,16,23,24	7,10,11,15,20,25	1,6,7,13,19,24	1,3,5,6,15,16
1,3,5,6,15,18	1,3,5,6,15,25	5,10,15,23,24,25	6,7,13,16,20,24	1,5,7,15,17,23	6,7,14,18,20,24
7,8,10,18,19,22	5,10,13,19,22,24	1,4,5,6,14,24	5,9,10,18,24,26	1,6,7,15,18,19	1,6,7,15,18,20
6,7,13,18,19,24	4,5,9,10,22,25	3,6,7,11,22,24	1,6,7,14,19,24	1,6,7,10,15,20	1,6,7,13,20,24
3,6,7,8,22,24	1,5,7,9,23,25	6,7,14,20,22,24	4,6,7,8,22,24	5,8,10,22,24,26	1,5,7,14,19,24
6,10,12,15,20,25	1,5,8,10,24,26	3,5,6,13,19,24	1,4,5,6,15,16	1,4,5,6,15,18	1,4,5,6,15,25
6,7,15,18,23,24	1,5,7,9,16,23	1,5,7,9,16,26	1,6,7,15,19,25	6,7,10,19,22,24	6,7,13,18,20,24
1,6,9,10,19,24	4,6,7,14,17,22	1,4,5,6,9,16	1,4,5,6,9,18	1,4,5,6,9,25	1,6,7,8,23,24
6,7,8,22,23,24	1,6,7,14,20,24	5,10,15,24,25,26	6,9,10,19,22,24	1,5,6,13,19,24	6,10,15,16,17,20
4,5,9,10,16,22	1,5,9,10,19,24	3,5,6,14,19,24	4,5,8,10,22,24	1,6,7,15,20,25	1,5,6,14,19,24
1,6,7,8,24,26	6,7,8,22,24,26	1,7,8,10,19,22	3,5,10,11,15,25	5,10,15,20,22,25	6,7,14,19,22,24
1,5,7,9,25,26	6,7,14,20,24,25	3,6,7,11,17,22	1,3,5,6,9,16	1,3,5,6,9,18	1,3,5,6,9,25
5,7,9,19,22,24	1,5,7,9,18,23	1,5,7,9,18,26	1,6,10,15,17,20	1,5,7,11,24,26	6,9,10,16,19,24
6,7,10,15,20,25	6,9,10,19,24,25	1,4,6,7,8,22	5,9,10,24,25,26	6,7,15,23,24,25	4,5,9,10,18,22
4,6,7,11,22,24	7,8,10,19,22,24	7,8,10,19,22,25	5,10,15,16,24,26	1,5,7,9,19,24	6,7,13,19,22,24
3,5,10,13,15,25	6,7,13,20,24,25	6,10,12,15,16,20	4,5,10,11,22,24	4,5,6,13,19,24	6,7,14,16,19,24
5,10,15,20,24,25					

Table 4. Solutions Requiring 3 Moves To Add Last Piece

5,7,16,19,22,24	4,5,9,10,19,22	4,5,10,15,19,22	6,8,11,12,18,24	6,8,12,15,17,19	6,7,15,17,20,26
5,6,12,22,24,26	7,8,9,13,15,19	6,8,12,14,18,24	3,6,7,8,19,22	7,8,9,12,16,22	7,8,11,13,15,20
4,6,7,8,19,22	5,10,15,19,23,24	1,5,6,12,24,26	7,12,13,14,15,20	1,5,6,18,19,24	4,5,9,10,20,22
4,5,7,13,22,24	7,11,12,13,15,20	5,10,15,19,24,26	1,3,5,6,15,19	1,5,7,19,21,24	4,6,8,11,12,15
4,5,7,9,22,23	4,5,7,9,22,26	1,4,5,7,13,22	4,5,7,15,22,26	6,7,12,15,20,26	3,5,9,10,19,22
7,8,9,11,13,15	7,8,9,11,13,24	5,6,18,19,22,24	4,6,8,12,13,15	1,5,7,15,19,23	7,8,11,12,20,22
7,8,12,14,20,22	6,7,15,19,23,24	7,8,11,12,13,22	3,5,9,10,20,22	3,5,10,15,19,22	7,9,11,12,13,15
7,9,11,12,13,24	5,10,15,19,20,24	1,5,7,16,19,22	4,5,7,21,22,24	7,8,12,13,14,22	7,8,9,12,21,24
4,6,8,12,14,15	7,8,13,14,15,20	3,6,8,9,11,12	6,7,15,19,24,26	4,6,8,9,11,12	7,8,9,11,15,20
7,8,12,13,15,16	6,8,9,12,18,24	7,8,12,13,15,25	4,5,6,15,24,26	7,9,12,13,15,19	6,8,11,12,17,24
6,8,12,15,16,17	3,6,8,9,12,13	3,6,8,9,12,14	3,6,8,9,12,15	4,6,8,9,12,13	4,6,8,9,12,14
4,6,8,9,12,15	7,8,9,12,15,23	7,8,9,12,15,26	6,8,12,13,18,24	7,8,11,12,15,16	7,8,11,12,15,25
7,9,11,12,15,20	7,8,12,14,15,21	7,8,12,14,15,23			

Table 5. Solutions Requiring 4 Moves To Add Last Piece

3,7,8,9,11,12	4,7,8,9,11,12	5,7,12,22,24,26	6,8,9,11,13,15	6,8,9,11,13,24	3,7,8,9,12,13
3,7,8,9,12,14	3,7,8,9,12,15	4,7,8,9,12,13	4,7,8,9,12,14	4,7,8,9,12,15	7,8,11,12,17,24
7,8,12,15,16,17	7,8,12,13,18,24	6,8,9,12,21,24	4,7,8,11,12,15	6,8,11,12,20,22	6,8,12,14,20,22
7,8,9,12,18,24	7,8,11,12,18,24	7,8,12,15,17,19	6,8,9,11,15,20	7,8,12,14,18,24	6,8,11,12,13,22
6,9,11,12,13,15	6,9,11,12,13,24	6,8,12,13,14,22	6,8,13,14,15,20	6,8,9,12,15,23	6,8,9,12,15,26
6,12,13,14,15,20	6,8,12,13,15,16	6,8,12,13,15,25	4,7,8,12,13,15	6,9,12,13,15,19	6,8,9,13,15,19
6,8,9,12,16,22	6,11,12,13,15,20	6,8,11,12,15,16	6,8,11,12,15,25	6,9,11,12,15,20	6,8,12,14,15,21
6,8,12,14,15,23	4,7,8,12,14,15	6,8,11,13,15,20			

Table 6. Solutions Requiring 5 Moves To Add Last Piece

6,8,9,12,19,25	7,8,9,12,16,19	7,8,9,12,16,20	6,8,12,13,19,22	6,8,12,13,19,24	6,8,9,12,20,25
6,8,11,12,19,24	6,8,12,13,20,24	6,8,12,15,19,20	6,8,12,15,19,21	6,8,11,12,20,24	7,8,9,12,18,19
7,8,9,12,18,20	7,8,12,13,19,22	7,8,12,13,19,24	6,8,12,15,20,21	7,8,9,12,19,25	7,8,11,12,19,24
7,8,12,13,20,24	6,8,12,13,15,26	6,8,9,12,16,19	6,8,9,12,16,20	7,8,9,12,20,25	7,8,12,15,19,20
7,8,12,15,19,21	7,8,11,12,20,24	6,8,11,12,15,26	7,8,12,15,20,21	6,8,9,12,18,19	6,8,9,12,18,20
7,8,12,13,15,26	7,8,11,12,15,26				